Sample Question Paper - 24
Mathematics-Standard (041)
Class- X, Session: 2021-22
TERM II

Time Allowed : 2 hours | *Maximum Marks : 40*

General Instructions :

1. *The question paper consists of 14 questions divided into 3 sections A, B, C.*
2. *All questions are compulsory.*
3. *Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.*
4. *Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.*
5. *Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.*

SECTION - A

1. If S_n, the sum of the first n terms of an A.P. is given by $S_n = 2n^2 + n$, then find its n^{th} term.

2. If the equation $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots then show that $c^2 = a^2(1+m^2)$.

3. Find the mean of the following distribution :

Class	3 - 5	5 - 7	7 - 9	9 - 11	11 - 13
Frequency	5	10	10	7	8

OR

The mean weight of a class of 35 students is 45 kg. If the weight of the teacher be included, the mean weight increases by 500 grams. Find the weight of the teacher.

4. 2 cubes, each of volume 125 cm^3, are joined end to end. Find the surface area of the resulting cuboid.

5. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that $OQ = 13$ cm. Find the length of PQ.

OR

A quadrilateral $ABCD$ is drawn circumscribing a circle with centre at O, the circle touches the sides AB, BC, CD and AD at P, Q, R and S respectively. If $\angle D = 90°$, $BC = 42$ cm, $CD = 30$ cm and $BP = 28$ cm. Find the radius of the circle.

6. What are the roots of the equation $x^2 + x - p(p + 1) = 0$,where p is a constant ?

SECTION - B

7. If the median of the distribution given below is 28.5, then find the values of x and y.

Class interval	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	Total
Frequency	5	x	20	15	y	5	60

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8. Draw a circle of radius 5 cm. From a point *P*, 8 cm away from its centre, construct a pair of tangents to the circle. Measure the length of each one of the tangents.

9. A person walking 20 m towards a chimney in a horizontal line through its base observes that its angle of elevation changes from 30° to 45°. Find the height of chimney.

OR

A man rowing a boat away from a lighthouse 150 m high takes 2 minutes to change the angle of elevation of the top of lighthouse from 45° to 30°. Find the speed of the boat. [Use $\sqrt{3}$ = 1.732]

10. In a class test, marks scored by students are given in the following frequency distribution :

Marks	0 - 6	6 - 12	12 - 18	18 - 24	24 - 30
Number of students	1	4	8	3	4

Find the mean and median of the data.

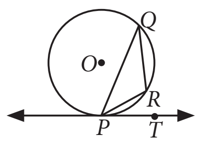
SECTION - C

11. The interior of a building is in the form of a cylinder of diameter 4.3 m and height 3.8 m, surmounted by a cone whose vertical angle is a right angle. Find the area of the surface and volume of the building.

OR

A cylindrical container of radius 4 cm and height 15 cm is filled with ice-cream. The whole ice-cream has to be distributed to 15 children in equal cones with hemispherical tops. If the height of the conical part is four times the radius of its base, then find the radius of the ice-cream cone.

12. In the given figure, *PQ* is a chord of a circle with centre *O* and *PT* is a tangent. If $\angle QPT = 60°$, then find $\angle PRQ$.



Case Study - 1

13. While playing a treasure hunt game, some clues (numbers) are hidden in various spots collectively forms an A.P. If the number on the n^{th} spot is $20 + 4n$, then answer the following questions to help the player in spotting the clues.

(i) Which number is on the $(n - 2)^{th}$ spot?

(ii) What is the sum of all the numbers on the first 10 spots?

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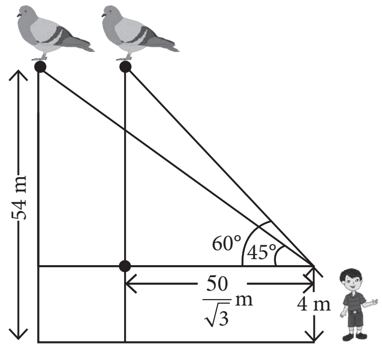
Case Study - 2

14. A boy 4 m tall spots a pigeon sitting on the top of a pole of height 54 m from the ground. The angle of elevation of the pigeon from the eyes of boy at any instant is 60°. The pigeon flies away horizontally in such a way that it remained at a constant height from the ground. After 8 seconds, the angle of elevation of the pigeon from the same point is 45°.



Based on the above information, answer the following questions. (Take $\sqrt{3}=1.73$)

(i) Find the distance of first position of the pigeon from the eyes of the boy.

(ii) How much distance the pigeon covers in 8 seconds?

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Solution

MATHEMATICS STANDARD 041

Class 10 - Mathematics

1. We have, $S_n = 2n^2 + n$

$\therefore$ $S_{n-1} = 2(n-1)^2 + (n-1) = 2(n^2 + 1 - 2n) + n - 1$

$= 2n^2 + 2 - 4n + n - 1 = 2n^2 - 3n + 1$

Now, n^{th} term of the A.P., *i.e.*, $a_n = S_n - S_{n-1}$

$= (2n^2 + n) - (2n^2 - 3n + 1) = 4n - 1$

2. We have, $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$

Since, this equation has equal roots.

$\therefore$ Discriminant, $D = 0$

$\Rightarrow (2mc)^2 - 4(1 + m^2)(c^2 - a^2) = 0$

$\Rightarrow 4m^2c^2 - 4c^2 + 4a^2 - 4m^2c^2 + 4a^2m^2 = 0$

$\Rightarrow c^2 = a^2 + a^2m^2 = a^2(1 + m^2)$

3. The frequency distribution table from the given data can be drawn as :

Class-interval	Frequency (f_i)	Class mark (x_i)	$f_i x_i$
3 - 5	5	4	20
5 - 7	10	6	60
7 - 9	10	8	80
9 - 11	7	10	70
11 - 13	8	12	96
Total	40		326

$\therefore$ Mean $= \dfrac{\Sigma f_i x_i}{\Sigma f_i} = \dfrac{326}{40} = 8.15$

OR

Mean weight of 35 students = 45 kg

Sum of weight of 35 students = 35 × 45 = 1575 kg

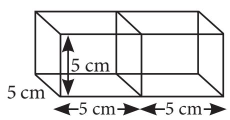
Let x kg be the weight of teacher.

$\therefore$ Mean weight $= \dfrac{1575 + x}{36}$

$\Rightarrow 45 + 0.5 = \dfrac{1575 + x}{36} \Rightarrow 1638 = 1575 + x \Rightarrow x = 63$

Hence, weight of teacher is 63 kg.

4. Let the edge of each cube be x cm.



$\therefore$ Volume of each cube $= x^3$ cm^3

i.e., $x^3 = 125 = (5)^3 \Rightarrow x = 5$

Thus, $l = 5 + 5 = 10$ cm, $b = 5$ cm and $h = 5$ cm.

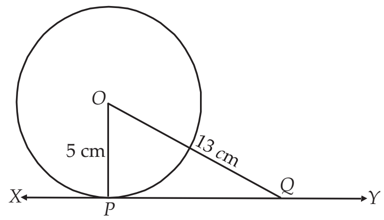
$\therefore$ Surface area of the cuboid

$= 2(lb + bh + hl) = 2(10 \times 5 + 5 \times 5 + 5 \times 10)$

$= 2(50 + 25 + 50) = 2 \times 125 = 250$ cm^2

5. Since, tangent at a point is perpendicular to the radius through that point. Therefore, OP is perpendicular to PQ.

In right angled triangle OPQ, we have



$OQ^2 = OP^2 + PQ^2 \Rightarrow 13^2 = 5^2 + PQ^2$

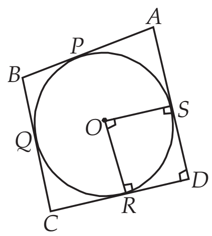
$\Rightarrow PQ^2 = 169 - 25 = 144 \Rightarrow PQ = 12$ cm.

OR

Given, $\angle D = 90°$, $BC = 42$ cm, $CD = 30$ cm and $BP = 28$ cm

Now $BP = BQ$ [$\because$ Two tangents drawn from an external point are equal]

Similarly, $CQ = CR$



$\therefore$ $BQ = 28$ cm

and $CQ = BC - BQ$

$= 42 - 28 = 14$ cm

or $CQ = CR$

$\therefore$ $CR = 14$ cm

$RD = CD - CR$

$= 30 - 14 = 16$ cm

Since $RD = OS$ = radius of the circle

$\therefore$ $OS = 16$ cm

Hence, radius of the circle is 16 cm.

6. Given equation is $x^2 + x - p(p + 1) = 0$

Using quadratic formula,

$$x = \frac{-1 \pm \sqrt{1 - (4)(-p^2 - p)}}{2}$$

$$= \frac{-1 \pm \sqrt{(2p+1)^2}}{2} = \frac{-1 \pm (2p+1)}{2}$$

$$\therefore \quad x = \frac{-1 + (2p+1)}{2} \quad \text{or} \quad x = \frac{-1 - (2p+1)}{2}$$

$$\Rightarrow \quad x = \frac{2p}{2} = p \quad \text{or} \quad x = \frac{-2 - 2p}{2} = -(p + 1)$$

7.

Class interval	Frequency	Cumulative frequency
0-10	5	5
10-20	x	$5 + x$
20-30	20	$25 + x$
30-40	15	$40 + x$
40-50	y	$40 + x + y$
50-60	5	$45 + x + y$
Total	$n = 60$	

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Here, we have $n = 60$

Since, median = 28.5

$\therefore$ Median class is 20 - 30 and

$l = 20, h = 10, f = 20, cf = 5 + x, n = 60$

$$\therefore \text{ Median} = l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$$

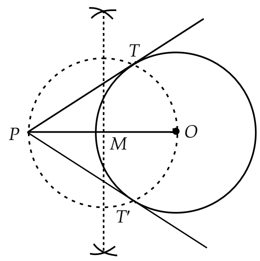
$$\Rightarrow 28.5 = 20 + \left[\frac{30 - (5 + x)}{20}\right] \times 10 \Rightarrow 28.5 = 20 + \frac{25 - x}{2}$$

$\Rightarrow 57 = 40 + 25 - x \Rightarrow x = 40 + 25 - 57 = 8$

Also, $45 + x + y = 60 \Rightarrow 45 + 8 + y = 60$

$\Rightarrow y = 60 - 45 - 8 = 7$. Thus $x = 8, y = 7$

8. Steps of construction :



Step-I : Draw a circle with O as centre and radius 5 cm.

Step-II : Mark a point P outside the circle such that $OP = 8$ cm.

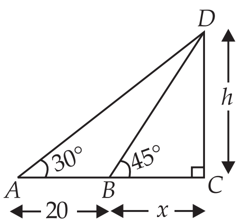
Step-III : Join OP and draw its perpendicular bisector, which cuts OP at M.

Step-IV : Draw a circle with M as centre and radius equal to MP to intersect the given circle at the point T and T'. Join PT and PT'.

Hence, PT and PT' are the required tangents.

9. Suppose height of the chimney is h metres. Let A and B be the point of observation and BC be x m.

In ΔACD,



$$\tan 30° = \frac{CD}{AC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{20 + x}$$

$$\Rightarrow 20 + x = h\sqrt{3}$$

$$\Rightarrow x = h\sqrt{3} - 20 \quad \text{...(1)}$$

Now, in ΔDBC, $\tan 45° = \dfrac{CD}{BC}$

$$\Rightarrow 1 = \frac{h}{x} \Rightarrow x = h \quad \text{...(2)}$$

From (1) and (2), we get

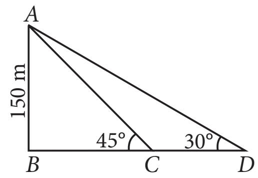
$$h = h\sqrt{3} - 20 \Rightarrow h\sqrt{3} - h = 20$$

$$\therefore h = \frac{20}{\sqrt{3} - 1} \text{ m}$$

OR

Let AB be the lighthouse of height 150 m.

Let initially boat is at C and after 2 minutes it reaches at D.



In ΔABC, $\dfrac{AB}{BC} = \tan 45°$

$$\Rightarrow \frac{150}{BC} = 1 \Rightarrow BC = 150 \text{ m}$$

In ΔABD, $\dfrac{AB}{BD} = \tan 30°$

$$\Rightarrow \frac{150}{BD} = \frac{1}{\sqrt{3}} \Rightarrow BD = 150\sqrt{3} \text{ m}$$

$\therefore$ Distance covered in 2 minutes is

$CD = BD - BC = 150\sqrt{3} - 150 = 150(\sqrt{3} - 1)$m

$$\therefore \text{ Speed of boat} = \frac{\text{Distance covered}}{\text{Time taken}} = \frac{150(\sqrt{3} - 1)}{2}$$

$= 75 \times (1.732 - 1) = 54.9$ m/minutes

10. We have the following table :

Marks	**Mid value (x_i)**	**Number of students (f_i)**	$f_i x_i$	**Cumulative frequency**
0–6	3	1	3	1
6–12	9	4	36	5
12–18	15	8	120	13
18–24	21	3	63	16
24–30	27	4	108	20
		$\Sigma f_i = 20$	$\Sigma f_i x_i = 330$	

$$\therefore \text{ Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{330}{20} = 16.5$$

Here, $\dfrac{n}{2} = \dfrac{20}{2} = 10$. The cumulative frequency just greater than 10 is 13 and its corresponding class is 12-18.

$\therefore l = 12, cf = 5, f = 8$ and $h = 6$

$$\therefore \text{ Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h = 12 + \frac{10 - 5}{8} \times 6$$

$$= 12 + \frac{15}{4} = 15.75$$

11. Radius of both the cylindrical and conical part, $r = \dfrac{4.3}{2} = 2.15$ m

Height of cylindrical part, $h = 3.8$ m

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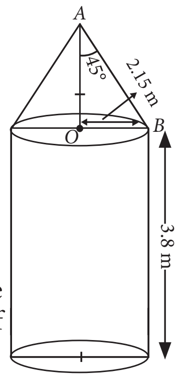
$\because$ Vertical angle of conical part = 90°

$\Rightarrow$ Semi-vertical angle of conical part = 45°



In ΔAOB, $\tan 45° = \dfrac{OB}{OA}$

$\Rightarrow 1 = \dfrac{2.15}{OA} \Rightarrow OA = 2.15$ m

$\therefore$ Height of cone, $H = 2.15$ m

Slant height of cone, $l = \sqrt{r^2 + H^2}$

$= \sqrt{(2.15)^2 + (2.15)^2} = 2.15\sqrt{2} = 3.04$ m

Surface area of building = Curved surface area of cone + Curved surface area of cylinder = $\pi rl + 2\pi rh$

$= \pi r[l + 2h]$

$= \dfrac{22}{7} \times 2.15[3.04 + 2(3.8)] = \dfrac{22}{7} \times 2.15 \times 10.64$

$= 71.90$ m^2

Volume of the building = Volume of the cylinder + Volume of the cone

$= \pi r^2 h + \dfrac{1}{3}\pi r^2 H = \pi r^2\left(h + \dfrac{1}{3}H\right)$

$= \dfrac{22}{7} \times (2.15)^2\left[3.8 + \dfrac{2.15}{3}\right] = \dfrac{22}{7} \times (2.15)^2 \times \dfrac{13.55}{3}$

$= 65.62$ m^3

OR

Let r be the radius of the base of the conical part and hemispherical part.

$\therefore$ Height of the conical part $(h) = 4r$

Volume of a cone with hemispherical top

= Volume of the conical part + Volume of the hemispherical part

$= \dfrac{1}{3}\pi r^2 h + \dfrac{2}{3}\pi r^3 = \dfrac{1}{3}\pi r^2 \times 4r + \dfrac{2}{3}\pi r^3 = \dfrac{6}{3}\pi r^3$

$= 2\pi r^3$ cm^3

Volume of 15 such cones = $15 \times 2\pi r^3 = 30\pi r^3$ cm^3

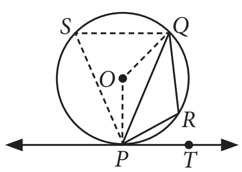
Also, volume of the cylindrical container = $\pi \times 4^2 \times 15 = 240\pi$ cm^3

$\because$ Volume of 15 cones with hemispherical tops = Volume of the cylindrical container

$\Rightarrow 30\pi r^3 = 240\pi \Rightarrow r^3 = 8 \Rightarrow r = 2$

Hence, radius of the ice-cream cone is 2 cm.

12. Join OP and OQ.



$\angle OPT = 90°$ ...(i) [$\because OP \perp PT$]

$\angle QPT = 60°$ ...(ii) [Given]

Subtracting (ii) from (i), we get

$\angle OPT - \angle QPT = 90° - 60°$

$\Rightarrow \angle OPQ = 30°$ ...(iii)

In ΔOPQ,

$OP = OQ$ [Radii of the same circle]

$\therefore \angle OQP = \angle OPQ$ [Angles opposite to equal sides of a triangle are equal]

$\Rightarrow \angle OQP = \angle OPQ = 30°$ [From (iii)] ...(iv)

In ΔOPQ, $\angle POQ + \angle OPQ + \angle OQP = 180°$ [By angle sum property of a triangle]

$\Rightarrow \angle POQ + 30° + 30° = 180°$ [Using (iv)]

$\Rightarrow \angle POQ + 60° = 180° \Rightarrow \angle POQ = 180° - 60° = 120°$

Now, let S be any point in the major segment PQ. Join PS and QS.

Now, $2\angle PSQ = 120°$ [$\because$ Angle subtended by an arc $\overset{\frown}{PQ}$ of a circle at the centre is twice the angle subtended by it at any point on the remaining part of the circle]

$\Rightarrow \angle PSQ = 60°$ ...(v)

Now, $SPRQ$ is a cyclic quadrilateral.

$\therefore \angle PSQ + \angle PRQ = 180° \Rightarrow 60° + \angle PRQ = 180°$ [Using (v)]

$\Rightarrow \angle PRQ = 120°$

13. Number on n^{th} spot = $20 + 4n$

i.e., $t_n = 20 + 4n$

(i) Number on $(n-2)^{th}$ spot = t_{n-2}

$= 20 + 4(n-2) = 20 + 4n - 8 = 12 + 4n$

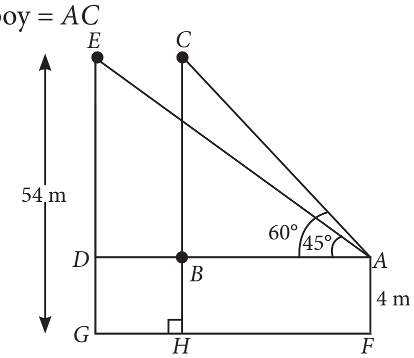
(ii) Here, $a = t_1 = 24$

Now, $t_2 = 20 + 4(2) = 20 + 8 = 28$

$\therefore d = t_2 - t_1 = 4$

So, required sum $= S_{10} = \dfrac{10}{2}[2(24) + 9(4)] = 420$

14. (i) Distance of first position of pigeon from the eyes of boy = AC



In ΔABC,

$\sin 60° = \dfrac{BC}{AC} \Rightarrow AC = \dfrac{CH - BH}{\sin 60°} = \dfrac{54 - 4}{\sqrt{3}/2} = \dfrac{100}{\sqrt{3}}$ m

(ii) In ΔAED, $\tan 45° = \dfrac{ED}{AD}$

$\Rightarrow AD = BC = 50$ m ($\because ED = BC$)

Now, distance between two positions of pigeon = EC

$= BD = AD - AB$

$= \left(50 - \dfrac{50}{\sqrt{3}}\right)$ m $= \dfrac{50(1.73 - 1)}{1.73} = 21.09$ m

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